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Thermally developing laminar flow inside rectangular ducts

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Abstract—Laminar forced convection inside rectangular ducts is analytically studied by extending the generalized integral transform technique, allowing for the solution of convection–diffusion problems with non-separable eigenvalue problems. Reference results are established for quantities of practical interest within the thermal entry region, for a wide range of the axial variable and various aspect ratios. The accuracy of previously reported results from direct numerical approaches is then critically examined, for both the developing and fully developed regions.

INTRODUCTION

HEAT TRANSFER solutions for laminar forced flow inside ducts of various shapes is of great interest to the design of compact heat exchangers, solar collectors and several other low Reynolds number flow heat exchange devices [1, 2]. The establishment of benchmark results through analytical solutions is quite desirable [2] for both reference purposes and validation of direct numerical schemes, especially for thermally developing flows. Except for the simpler situations of ducts with cross sections definable by a single coordinate, such as circular tubes, parallel-plate channels, and annular ducts, a very limited amount of analytical work is available in the literature, and most contributions deal with purely numerical or approximate approaches [1]. The case of a rectangular duct is a typical example of the difficulties associated with solving multidimensional convection problems, requiring costly numerical solutions [1], limited to regions away from the inlet (longer ducts). The exact solution of such a problem, through classical analytical methods [3] is inhibited due to the non-separable nature of the related eigenvalue problem. The present contribution attempts to alleviate this difficulty by extending the ideas in the so-called generalized integral transform technique [4–11] to allow for the solution of this formally transformable but non-separable problem, providing an efficient algorithm for numerical computations.

The case of a rectangular duct subjected to a constant wall temperature is more closely considered to illustrate the approach. An analysis of convergence is made and a set of benchmark results established for quantities of practical interest, such as dimensionless average temperature, local and average Nusselt numbers, within a wide range of the dimensionless axial coordinate. Previously reported results [1, 13–15] from direct numerical approaches are then critically examined for both fully developed and thermally developing regions, limited to a narrow range of the dimensionless axial coordinate.

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ANALYSIS

We consider laminar flow of a Newtonian fluid inside a rectangular channel of sides $2a$ and $2b$, with a fully developed velocity profile and subjected to a constant wall temperature. For thermally developing flow the associated energy equation is written in dimensionless form as

$$U(X,Y)\frac{\partial\theta(X,Y,Z)}{\partial Z}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2},$$

$$0<X<\alpha,\quad 0<Y<\beta,\quad Z>0 \qquad (1a)$$

with inlet and boundary conditions given, respectively, by

$$\theta(X,Y,0)=1,\quad 0\leqslant X\leqslant\alpha,\quad 0\leqslant Y\leqslant\beta \qquad (1b)$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=0}=0;\quad \theta(\alpha,Y,Z)=0,\quad Z>0 \qquad (1c,d)$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=0}=0;\quad \theta(X,\beta,Z)=0,\quad Z>0 \qquad (1e,f)$$

where the following dimensionless groups were defined:

$$\theta(X,Y,Z)=\frac{T(x,y,z)-T_w}{T_i-T_w};\quad X=\frac{x}{D_h};\quad Y=\frac{y}{D_h}$$

$$U(X,Y)=\frac{u(x,y)}{u_m};\quad Z=\frac{z}{D_h\,Pe};\quad Pe=\frac{\rho c_p}{K}u_m D_h$$

$$\alpha=\frac{a}{D_h};\quad \beta=\frac{b}{D_h}. \qquad (2)$$

NOMENCLATURE

a	half the side of the rectangular duct in the x-direction
A_c	cross-sectional area
b	half the side of the rectangular duct in the y-direction
D_h	hydraulic diameter, $4A_c/P$
$h(z)$	local heat transfer coefficient
$h_{av}(z)$	average heat transfer coefficient
K	thermal conductivity
Pe	Peclet number, $Re\ Pr$
r_j	coefficients from problem (15b), $j = 1, \dots, N \cdot N^*$
T_i	inlet temperature
T_w	wall temperature
u_m	average flow velocity
$u(x, y)$	velocity field
x	transversal coordinate
y	transversal coordinate
z	axial coordinate.

Greek symbols

α^*	aspect ratio of rectangular duct, $2b/2a$
γ_j	eigenvalues of matrix F, as in problem (14)
$\zeta^{(j)}$	eigenvector of matrix F, as in problem (14)
λ_m	eigenvalues of problem (5)
μ_i	eigenvalues of problem (4)
$\phi_m(y)$	eigenfunctions of problem (5)
$\psi_i(X)$	eigenfunctions of problem (4).

The dimensionless velocity profile is given as an infinite series [1]

$$U(X, Y) = A^*(\alpha^*) \sum_{k=1,3,\dots}^{\infty} B_k F_k(Y) G_k(X) \quad (3a)$$

where

$$A^*(\alpha^*) = \frac{48}{\pi^3 \left\{1 - \frac{192}{\pi^5 \alpha^*} \sum_{k=1,3,\dots}^{\infty} \frac{\tanh(k\pi\alpha^*/2)}{k^5}\right\}} \quad (3b)$$

$$B_k = \frac{(-1)^{(k-1)/2}}{k^3} \quad (3c)$$

$$F_k(Y) = 1 - \frac{\cosh\left(\frac{k\pi Y}{2\alpha}\right)}{\cosh\left(\frac{k\pi\alpha^*}{2}\right)} \quad (3d)$$

$$G_k(X) = \cos\left(\frac{k\pi X}{2\alpha}\right) \quad \text{and}$$

$$\alpha^* = 2b/2a = \text{aspect ratio.} \quad (3e,f)$$

The exact solution of problem (1) through well-known analytical methods, such as the classical integral transform technique [3], is not possible due to the non-separable nature of the velocity profile and consequently, of the related eigenvalue problem. However, the recently advanced ideas on the generalized integral transform technique [4–11] can be modified to allow for an analytical treatment of the present problem as now demonstrated. First, the difficulties associated with the eigenvalue problem are alleviated by choosing the following auxiliary problems instead:

$$\frac{d^2\psi(\mu, X)}{dX^2} + \mu^2\psi(\mu, X) = 0; \quad 0 < X < \alpha \quad (4a)$$

$$\left.\frac{d\psi(\mu, X)}{dX}\right|_{X=0} = 0; \quad \psi(\mu, \alpha) = 0 \quad (4b,c)$$

and

$$\frac{d^2\phi(\lambda, Y)}{dY^2} + \lambda^2\phi(\lambda, Y) = 0; \quad 0 < Y < \beta \quad (5a)$$

$$\left.\frac{d\phi(\lambda, Y)}{dY}\right|_{Y=0} = 0; \quad \phi(\lambda, \beta) = 0 \quad (5b,c)$$

which are readily solved to yield eigenfunctions, eigenvalues, and norms as

$$\psi_i(x) = \cos(\mu_i X); \quad \phi_m(Y) = \cos(\lambda_m Y) \quad (6a,b)$$

$$\mu_i = \frac{(2i-1)\pi}{2\alpha}; \quad \lambda_m = \frac{(2m-1)\pi}{2\beta} \quad (6c,d)$$

$$N_i = \frac{\alpha}{2}; \quad M_m = \frac{\beta}{2}, \quad i, m = 1, 2, \dots, \quad (6e,f)$$

Problems (4) and (5) above allow the establishment of the following integral transform pair.

Transform

$$\bar{\theta}_{im}(Z) = \int_0^{\alpha}\int_0^{\beta} \psi_i(X)\phi_m(Y)\theta(X, Y, Z)\, dY\, dX. \quad (7a)$$

Inversion

$$\theta(X, Y, Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} \frac{\psi_i(X)\phi_m(Y)}{N_i M_m}\bar{\theta}_{im}(Z). \quad (7b)$$

Equation (1a) is now operated on with

$$\int_0^{\alpha}\int_0^{\beta} \psi_i(X)\phi_m(Y)\, dY\, dX$$

to yield, after employing the inversion formula above

$$\sum_{j=1}^{\infty}\sum_{n=1}^{\infty} D_{ijmn}\frac{\mathrm{d}\bar{\theta}_{jn}(Z)}{\mathrm{d}Z}+(\mu_i^2+\lambda_m^2)\bar{\theta}_{im}(Z)=0, \quad Z>0 \tag{8a}$$

where

$$D_{ijmn}=\frac{1}{N_iM_n}\int_0^{\alpha}\int_0^{\beta}\psi_i(X)\psi_j(X)\phi_m(Y)\phi_n(Y) \times U(X,Y)\,\mathrm{d}Y\,\mathrm{d}X \tag{8b}$$

while the transformed inlet condition becomes

$$\bar{\theta}_{im}(0)=\bar{g}_{im}=\frac{(-1)^{i+m}}{\mu_i\lambda_m}. \tag{8c}$$

The double integral in equation (8b) is evaluated to provide

$$D_{ijmn}=\frac{A^*(\alpha^*)}{N_jM_n}\sum_{k=1,3,\ldots}^{\infty} B_k\psi_{ijk}\psi^*_{mnk} \tag{9a}$$

where

$$\psi_{ijk}=\frac{(-1)^{i+j+(k-1)/2}}{4}\left\{\frac{1}{\mu_i+\mu_j-a_k}+\frac{1}{\mu_i-\mu_j+a_k}+\frac{1}{\mu_j-\mu_i+a_k}-\frac{1}{\mu_i+\mu_j+a_k}\right\} \tag{9b}$$

$$\psi^*_{mnk}=M_m\delta_{mn}+\frac{(-1)^{m+n+1}}{2}a_k\tanh(a_k\beta)\cdot\left\{\frac{1}{a_k+(\lambda_m-\lambda_n)^2}-\frac{1}{a_k+(\lambda_m+\lambda_n)^2}\right\}$$

$$a_k=\frac{k\pi}{2\alpha}. \tag{9c}$$

System (8) above provides a denumerable system of coupled ordinary differential equations with constant coefficients, to be solved for the transformed potentials, $\bar{\theta}_{im}$'s. So far the analysis is formal and exact, but for the sake of obtaining numerical results from this, *a priori*, formal solution, the infinite system (8) has to be truncated to a sufficiently large finite order for the desired convergence. Then, following ref. [8], the truncated system is written as

$$\sum_{j=1}^{N}\sum_{n=1}^{N^*} D_{ijmn}\frac{\mathrm{d}\bar{\theta}_{jn}(Z)}{\mathrm{d}Z}+(\mu_i^2+\lambda_m^2)\bar{\theta}_{im}(Z)=0,$$

$$i=1,2,\ldots,N,\quad m=1,2,\ldots,N^* \tag{10a}$$

with

$$\bar{\theta}_{im}(0)=\bar{g}_{im} \tag{10b}$$

and the finite system (10) of $N\cdot N^*$ coupled equations is given in matrix form

$$C\mathbf{y}'+E\mathbf{y}=\mathbf{0} \tag{11a}$$

$$\mathbf{y}(0)=\mathbf{g} \tag{11b}$$

where

$$\mathbf{y}=\{\bar{\theta}_{11}(Z),\bar{\theta}_{12}(Z),\ldots,\bar{\theta}_{1N^*}(Z),\bar{\theta}_{21}(Z),\ldots,\bar{\theta}_{2N^*}(Z),\ldots,\bar{\theta}_{N1}(Z),\ldots,\bar{\theta}_{NN^*}(Z)\}^{\mathrm{T}} \tag{12a}$$

$$\mathbf{g}=\{\bar{g}_{11},\bar{g}_{12},\ldots,\bar{g}_{1N^*},\bar{g}_{21},\ldots,\bar{g}_{2N^*},\ldots,\bar{g}_{N1},\ldots,\bar{g}_{NN^*}\}^{\mathrm{T}} \tag{12b}$$

$$E=\{e_{jn}\},\quad e_{jn}=\delta_{jn}(\mu_l^2+\lambda_k^2),\quad \delta_{jn}=\begin{cases}0, & i\neq j\\ 1, & i=j\end{cases} \tag{12c}$$

$$j,n=1,2,\ldots,N\cdot N^*;\quad l=\mathrm{int}\,(j/N^*)+1;$$
$$k=j-N^*\,\mathrm{int}\,((j-1)/N^*);\quad C=\{c_{jn}\};$$
$$c_{jn}=D_{ll'kk'};\quad j,n=1,2,\ldots,N\cdot N^*;$$
$$l'=\mathrm{int}\,(n/N^*)+1;$$
$$k'=n-N^*\,\mathrm{int}\,((n-1)/N^*). \tag{12d}$$

System (1) can then be rewritten in normal form through inversion of matrix C to yield

$$\mathbf{y}'+F\mathbf{y}=\mathbf{0} \tag{13a}$$

$$\mathbf{y}(0)=\mathbf{g} \tag{13b}$$

where

$$F=C^{-1}E. \tag{13c}$$

This finite $(N\cdot N^*)$ system with a constant coefficients matrix, F, can be readily solved once the eigenvalues and eigenvectors of the real matrix F have been obtained from the solution of the algebraic problem

$$(F-\gamma I)\zeta=0 \tag{14}$$

and the solution of system (13) is written as

$$\mathbf{y}(Z)=r_1\,\mathrm{e}^{-\gamma_1 Z}\zeta^{(1)}+\cdots+r_{N\cdot N^*}\,\mathrm{e}^{-\gamma_{N\cdot N^*}Z}\zeta^{(N\cdot N^*)} \tag{15a}$$

where the coefficients $r_1,\ldots,r_{N\cdot N^*}$ are obtained from satisfying the transformed inlet condition, which corresponds to solving the following system of linear algebraic equations:

$$\zeta^{(1)}r_1+\cdots+r_{N\cdot N^*}\zeta^{(N\cdot N^*)}=\mathbf{g}. \tag{15b}$$

Problems (14) and (15b) are handled by making use of well-known algorithms already available as packed subroutines, such as in ref. [12], to provide the transformed potentials from equation (15a). Alternatively, efficient numerical algorithms for initial value problems can be utilized to directly solve system (13), such as in subroutine DGEAR from the IMSL package [12], with high accuracy.

Once the transformed potentials have been obtained, the inversion formula (7b) is recalled to provide the complete temperature profile

$$\theta(X,Y,Z)=\frac{4}{\alpha\beta}\sum_{i=1}^{N}\sum_{m=1}^{N^*}\cos(\mu_iX)\cos(\lambda_mY)\bar{\theta}_{im}(Z). \tag{16}$$

The dimensionless average temperature is then computed from its definition

$$\theta_{av}(Z) = \frac{1}{A_c}\int_{A_c} U(X,Y)\theta(X,Y,Z)\,dA \quad (17a)$$

or

$$\theta_{av}(Z) = \frac{2A^*(\alpha^*)}{\alpha\beta^2}\sum_{i=1}^{N}\sum_{m=1}^{N^*} B_i Q_{im}\tilde{\bar{\theta}}_{im}(Z) \quad (17b)$$

where

$$Q_{im} = \frac{(-1)^{m+1}a_i^2}{\lambda_m(a_i^2+\lambda_m^2)}. \quad (17c)$$

The local Nusselt number can be evaluated by making use of the temperature gradients at the wall integrated over the perimeter, or utilizing the axial gradient of the average temperature, providing the following couple of working formulae:

$$Nu_1(Z) = \frac{h(z)D_h}{K} = -\frac{1}{(\alpha+\beta)\theta_{av}(Z)} \times\left\{\int_0^{\alpha}\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=\beta} dX + \int_0^{\beta}\left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=\alpha} dY\right\} \quad (18a)$$

$$Nu_2(Z) = \frac{h(z)D_h}{K} = -\frac{1}{4\theta_{av}(Z)}\frac{d\theta_{av}(Z)}{dZ} \quad (18b)$$

or

$$Nu_1(Z) = \frac{-2\beta}{(\alpha+\beta)A^*(\alpha^*)} \cdot \frac{\sum_{i=1}^{N}\sum_{m=1}^{N^*}\frac{(-1)^{i+m}}{\mu_i\lambda_m}(\mu_i+\lambda_m)\tilde{\bar{\theta}}_{im}(Z)}{\sum_{i=1}^{N}\sum_{m=1}^{N^*} B_i Q_{im}\tilde{\bar{\theta}}_{im}(Z)} \quad (18c)$$

$$Nu_2(Z) = -\frac{1}{4}\frac{\sum_{i=1}^{N}\sum_{m=1}^{N^*} B_i Q_{im}\frac{d\tilde{\bar{\theta}}_{im}}{dZ}}{\sum_{i=1}^{N}\sum_{m=1}^{N^*} B_i Q_{im}\tilde{\bar{\theta}}_{im}(Z)}. \quad (18d)$$

For fully converged $\tilde{\bar{\theta}}_{im}$'s, expressions (18c) and (18d) should yield the same numerical result. Therefore, the comparison of these results for decreasing Z provides an interesting check of convergence behavior. The average Nusselt numbers are then computed from

$$Nu_{av,1}(Z) = \frac{h_{av}(z)D_h}{K} = \frac{1}{Z}\int_0^Z Nu_1(Z)\,dZ \quad (19a)$$

$$Nu_{av,2}(Z) = \frac{h_{av}(z)D_h}{K} = \frac{1}{Z}\int_0^Z Nu_2(Z)\,dZ \quad (19b)$$

and equation (19b) can be analytically integrated to yield

$$Nu_{av,2}(Z) = -\frac{1}{4Z}\ln\theta_{av}(Z). \quad (19c)$$

RESULTS AND DISCUSSION

System (13) was solved with $N = N^* \leqslant 20$ to illustrate the convergence behavior of the present approach, within a wide range of Z, from 10^{-4} to 10^0. The dimensionless average temperature demonstrated excellent convergence characteristics, with practically coincident results for $N = N^* = 11$, 15 and 20. Figure 1 shows a comparison of $Nu_1(Z)$ and $Nu_2(Z)$, as obtained from equations (18c) and (18d), for a square duct ($\alpha^* = b/a = 1$). For $N = N^* = 11$ the results from these two expressions are practically coincident for $Z \geqslant 6\times10^{-4}$, with $N = N^* = 15$ for $Z \geqslant 2\times10^{-4}$, and with $N = N^* = 20$ for $Z \geqslant 10^{-4}$. Clearly, it can be noticed that Nusselt numbers computed from the heat balance, equation (18d), have better convergence than those from the application of Fourier's law, equation (18c). Figure 2 brings a comparison among numerical approaches [1, 13–15], in the range $10^{-2} < Z < 10^{-1}$. Apparently, the results in ref. [15] are the most accurate, while the variational solution in ref. [13] and the results in ref. [1] lose adherence as Z is decreased although not completely evident in the range presented. It was also observed that few terms are required ($N = N^* \simeq 5$) to obtain a converged Nusselt number within this range of Z considered by most numerical approaches. Table 1 presents a comparison of limiting Nusselt numbers ($Z \to \infty$) from various sources, compiled in ref. [1], including the following proposed correlation, which attempts to approximate the results of Miles and Shih [1] to within $\pm0.1\%$:

$$Nu(\infty) = 7.541(1-2.610\alpha^{*-1}+4.970\alpha^{*-2} -5.119\alpha^{*-3}+2.702\alpha^{*-4}-0.548\alpha^{*-5}) \quad (20)$$

where $Nu(\infty) = 7.541$ corresponds to the case of a parallel-plate channel ($\alpha^* \to \infty$). The results of Miles and Shih [1] with a 40×40 finite difference grid appear to be more accurate than those by Schmidt [1], and correlation (20) is sufficiently accurate for most practical purposes.

Figures 3–5 correspond, respectively, to the dimensionless average temperature, local Nusselt number, and average Nusselt number along the thermal entry region ($10^{-4} < Z < 10^0$), for different aspect ratios, α^*, providing a set of benchmark results both for reference purposes and calibration of purely numerical schemes devised for more involved problems.

Besides its efficiency, the present approach demonstrated to be relatively cheap, in the range of N and N^* considered. By incorporating the ideas in ref. [9] the present analysis might be extendable to the case of irregularly shaped duct geometries described by an orthogonal coordinate system.

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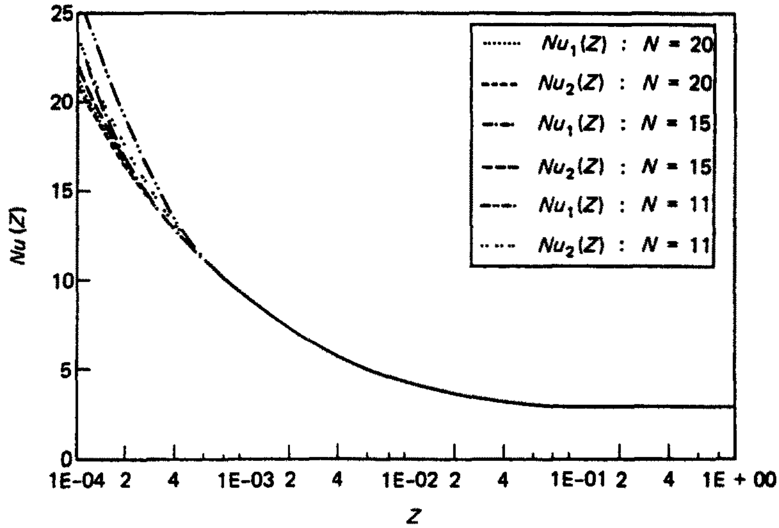

FIG. 1. Convergence of local Nusselt number for a square duct ($\alpha^* = b/a = 1$).

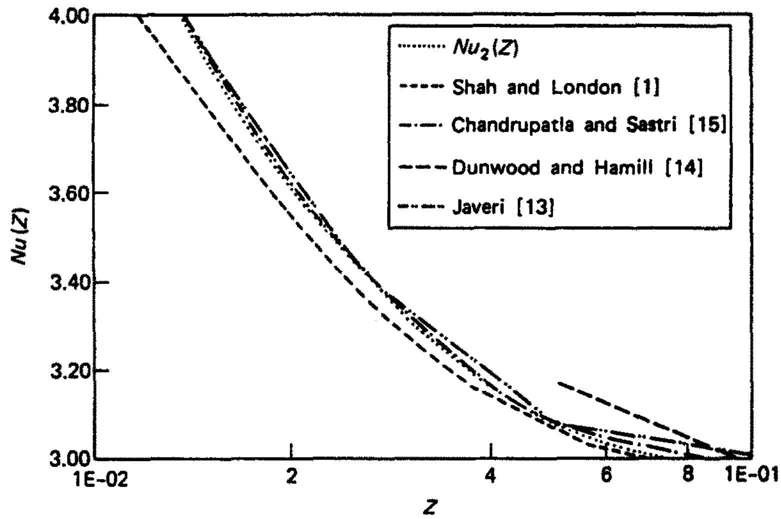

FIG. 2. Comparison with literature results from numerical approaches for a square duct ($\alpha^* = b/a = 1$).

Table 1. Comparison of limiting Nusselt numbers from different sources and for various aspect ratios

$\alpha = 2b/2a$	Exact†	Miles and Shih [1]	Schmidt [1]	Shah and London [1]‡
1.0	2.978	2.976	2.970	2.979
1.1	2.986	—	—	2.971
1.2	3.007	—	—	2.992
1.5	3.128	3.117	—	3.120
2.0	3.392	3.391	3.383	3.389
3.0	3.958	3.956	—	3.950
4.0	4.440	4.439	—	4.435
5.0	4.810	—	4.803	4.826
6.0	5.143	5.137	—	5.138
8.0	5.607	5.597	—	5.596
10.0	5.930	—	5.858	5.911

† Present solution.
‡ Equation (20).

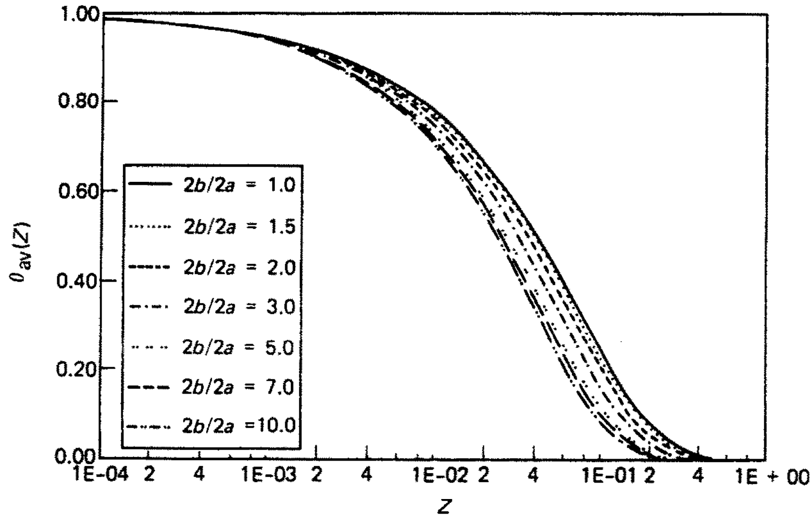

FIG. 3. Dimensionless average temperature profiles for rectangular ducts with different aspect ratios.

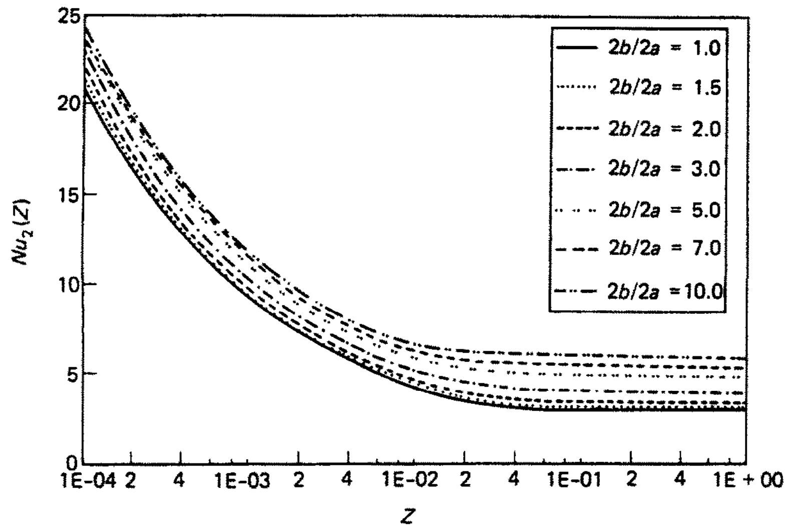

FIG. 4. Local Nusselt numbers in the thermal entry region of rectangular ducts with different aspect ratios.

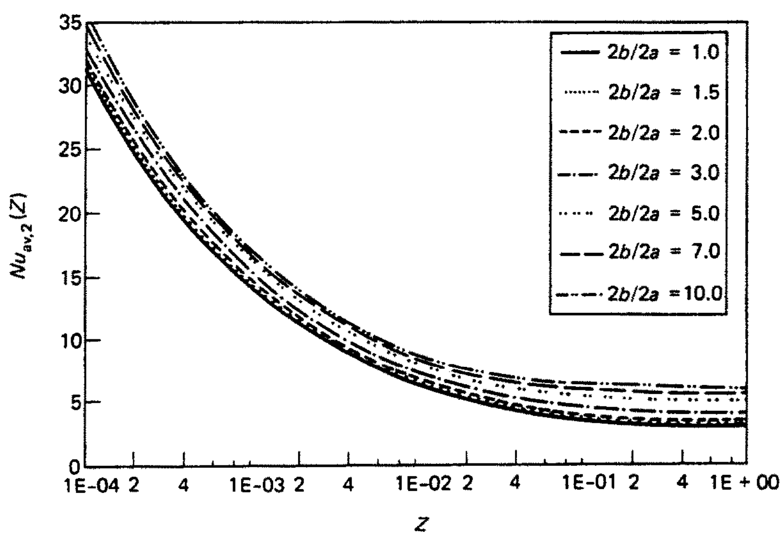

FIG. 5. Average Nusselt numbers in the thermal entry region of rectangular ducts with different aspect ratios.

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ECOULEMENT LAMINAIRE THERMIQUEMENT ETABLI A L'INTERIEUR DE CONDUITS RECTANGULAIRES

Résumé—La convection forcée laminaire dans des conduits rectangulaires est étudiée analytiquement, par extension de la technique générale de la transformation intégrale permettant la résolution des problèmes de convection–diffusion avec les problèmes de valeurs propres non séparables. Des résultats sont donnés pour des grandeurs d'intérêt pratique dans la région d'entrée, pour un large domaine des variables axiales et différents rapports de forme. La précision des résultats antérieurement obtenus par des approches numériques sont examinés de façon critique, à la fois pour la région en développement et pour la région établie.

THERMISCH NICHT AUSGEBILDETE STRÖMUNG IN RECHTECKIGEN KANÄLEN

Zusammenfassung—Die erzwungene laminare Konvektion im Innern eines rechteckigen Kanals wird analytisch untersucht. Die verallgemeinerte Integral-Transformationstechnik wird erweitert und bietet jetzt die Möglichkeit, Konvektions- und Diffusionsprobleme mit nicht separierbaren Eigenwerten zu lösen. Für die Größen, die im thermischen Einlaufgebiet von praktischem Interesse sind, stehen Referenzergebnisse über einen weiten Bereich von axialen Variablen und verschiedenen Abmessungsverhältnissen zur Verfügung. Die Genauigkeit früherer Ergebnisse bei direkter numerischer Berechnung wird kritisch untersucht, sowohl für das Einlaufgebiet als auch bei thermisch ausgebildeter Strömung.

ТЕРМИЧЕСКИ РАЗВИВАЮЩЕЕСЯ ЛАМИНАРНОЕ ТЕЧЕНИЕ В ПРЯМОУГОЛЬНЫХ КАНАЛАХ

Аннотация—Аналитически исследуется вынужденная ламинарная конвекция в прямоугольных каналах с применением обобщенного метода интегральных преобразований к решениям задач конвекции и диффузии и связанных с ними задач на собственные значения. Определены характерные значения параметров областей теплового входа, аксиальных переменных и различных соотношений сторон. Критически оценивается точность ранее опубликованных результатов, полученных прямыми численными методами для случаев развивающегося и полностью развитого участков.